Design and Analysis of an RLC Circuit for Radio Receiver Applications

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1 Introduction

This project explores the design and analysis of an RLC circuit tuned for radio receiver applications. The main objective is to calculate the resonant frequency, cutoff frequencies, and quality factor (Q) of the circuit. Additionally, we analyze the frequency response to assess the performance of the RLC circuit as part of the radio receiver design.

A design goal of this project is to accept a frequency of 88.7 MHz, but achieve a power reduction to $\frac{1}{4}$ of the original power for 88.5 MHz.

2 Methodology and Results

2.1 Resonant Frequency and Cutoff Frequencies

To achieve the desired resonance and cutoff frequencies:

- The resonant frequency was set to $f_{\text{res}} = 88.7$ MHz.
- Inductance was fixed at $L = 8.22\ \mu H$.
- Using these parameters, capacitance (C) and resistor (R) values were calculated to ensure the circuit's performance. The results are:

$$C = 3.9167 \times 10^{-13}\ \text{F}, \quad R = 11.941\ \Omega$$

- The upper and lower angular cutoff frequencies were:

$$\omega_L = 88.5848\ \text{MHz}, \quad \omega_H = 88.8155\ \text{MHz}.$$

Figure 1 shows the RLC series circuit used in this analysis, while Figure 2 provides the LTSpice simulation of its frequency response.

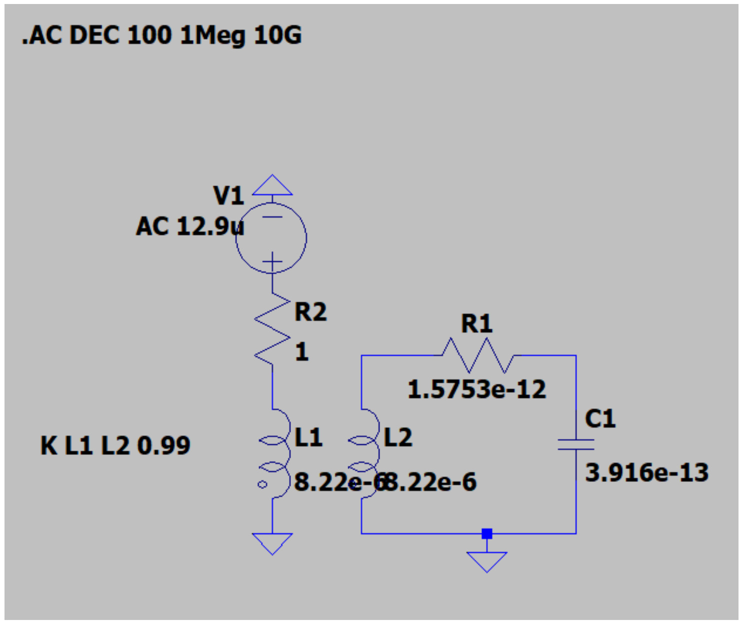

Figure 1: RLC Series Circuit Schematic.

2.2 Frequency Response

The frequency response of the circuit was analyzed using MATLAB and LT-Spice. Figure 3 shows the MATLAB-generated magnitude response $|H|$ of the RLC circuit across the frequency range. Key observations include:

- The circuit achieves a peak response at the resonant frequency of 88.7 MHz.
- The calculated bandwidth (BW) was 0.23072 MHz, with a quality factor of $Q = 384.4436$.
- The power ratio at 88.5 MHz was calculated as 0.25 relative to the resonance.

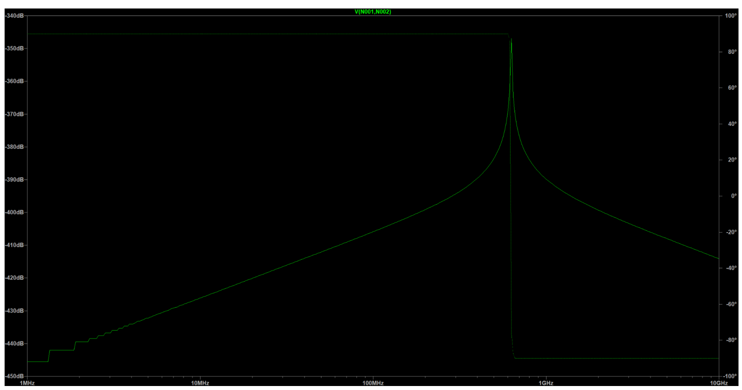

Figure 2: Frequency Response of the RLC Circuit Simulated in LTSpice.

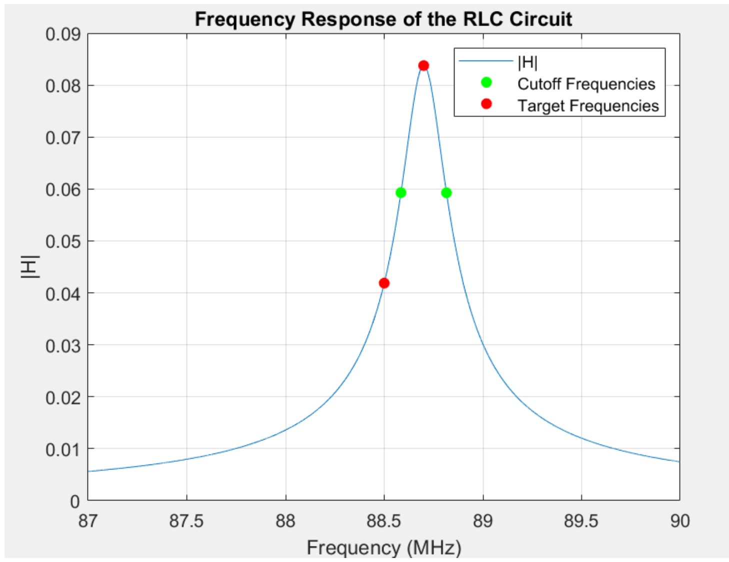

Figure 3: Frequency Response of the RLC Circuit Analyzed with MATLAB.

2.3 MATLAB Code

MATLAB was used to automate the calculations and visualize results. The scripts calculate:

- Resonant frequency capacitance and resistance for target cutoff frequencies.
- Bandwidth, quality factor, and frequency response.
- Visual representation of the circuit's performance.

Note: The complete code is available on my GitHub repository.

3 Mathematical Derivations

This section includes mathematical derivations based on Kirchhoff's Voltage Law (KVL), impedance analysis, and transfer function computations for the RLC circuit.

3.1 Derivation of Transfer Function

For a voltage V_R across the resistor R, the input voltage V_{in} can be expressed as:

$$V_{\text{in}} = E\cos(\omega t), \quad E_R = E - V_R. \tag{1}$$

Applying KVL:

$$E = IZ, \quad Z = R - j\left(\omega L - \frac{1}{\omega C}\right). \tag{2}$$

The current I is:

$$I = \frac{E}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}. \tag{3}$$

The transfer function, defined as the ratio of the voltage across the resistor V_R to the input voltage V_{in}, is given by:

$$H(j\omega) = \frac{V_R}{V_{\text{in}}} = \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}. \tag{4}$$

3.2 Condition for Power Reduction to 1/4

To find the resistor value R that makes the power drop to 1/4 of its original value at $\omega = 2\pi \cdot 88.5$ MHz, we use the following relationship:

$$\frac{R^2}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \frac{1}{4}. \tag{5}$$

Rewriting:

$$R^2 = \frac{1}{4}\left(R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right). \tag{6}$$

Simplify:

$$4R^2 = R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2. \tag{7}$$

$$3R^2 = \left(\omega L - \frac{1}{\omega C}\right)^2. \tag{8}$$

Solving for R:

$$R = \sqrt{\frac{\left(\omega L - \frac{1}{\omega C}\right)^2}{3}}. \tag{9}$$

3.3 Derivation of Quality Factor (Q)

The quality factor Q relates the resonant frequency ω_0 to the bandwidth BW:

$$Q = \frac{\omega_0}{BW}, \quad \omega_0 = \frac{1}{\sqrt{LC}}. \tag{10}$$

Using the cutoff frequencies ω_L and ω_H, we define BW:

$$BW = \omega_H - \omega_L. \tag{11}$$

At the cutoff points, the impedance condition $\sqrt{2}R = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$ is derived from the relationship between power and impedance:

- At the cutoff frequency, the power delivered to the resistor is reduced to half of its maximum value.
- Power is proportional to the square of the voltage across the resistor: $P \propto V_R^2$.
- For the power to halve, the voltage across the resistor must drop by a factor of $1/\sqrt{2}$: $|V_R| = |V_{\max}|/\sqrt{2}$.
- Since the transfer function magnitude $H(j\omega)$ describes $|V_R|/|V_{\text{in}}|$, this translates to the condition:

$$\sqrt{2}R = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}. \tag{12}$$

From this, we derive the limits for ω_L and ω_H:

$$\omega_L = \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}, \quad \omega_H = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}. \tag{13}$$

3.4 Summary

This analysis demonstrates the mathematical relationships for calculating the transfer function, quality factor, and cutoff frequencies in an RLC circuit.

4 Conclusion

Future work includes integrating these components into a functional radio receiver and further exploring AM demodulation. This is a current work in progress, as I am studying RF circuit design on my own this semester.